Programming contests tutorial

or

how to improve when you're already decent

Przemek Uznański

June 28, 2017

About myself

from University of Wrocław, Poland

Programming competitions: 2003-2013

- ACM ICPC,
- Google Code Jam,
- Facebook Hacker Cup,
- TopCoder

Coach of highschoolers (IOI) in Wrocław:

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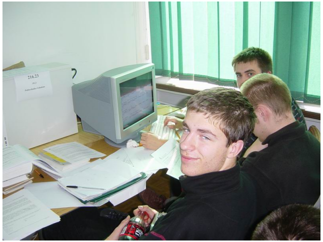

(2004, my first onsite)

Coach of highschoolers (IOI) in Wrocław:

End Goal:

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The Fortieth Annual
ACM-ICPC
World Finals
May 19, 2016
Phuket Thailand


TCO16
topcoder
cisco
docker
rng_58
MAKOTO SOEJIMA

How to get better?

I asked a few friends: “Any tips/tricks for getting better?”

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Practice, practice, practice. A lot.

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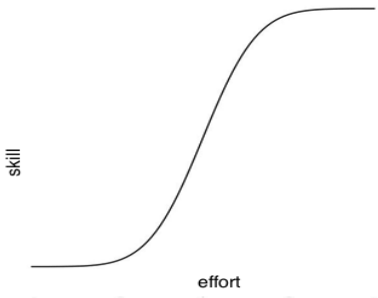
Plan for today:

- how to train effectively
- advanced topics:
  - number theory
  - hashing
- tips&tricks

How to train effectively?



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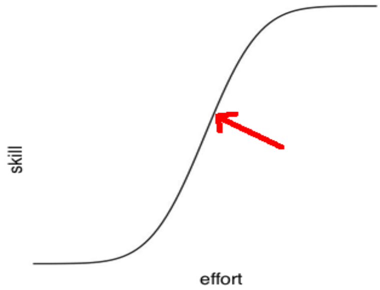

Diminishing returns:

lot of effort, little improvement

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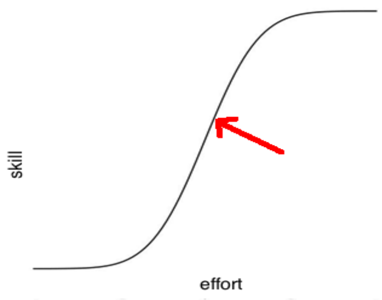

How to solve problems.

vs.

How to approach whole classes of problems.

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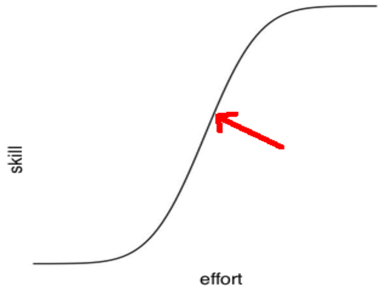

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Diminishing returns:
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Do not try to rediscover the wheel!

Number theory

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Typical tasks:

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- primality testing
- functions: $\varphi(n)$ (Euler's totient function), $\sigma(n)$ (sum of divisors), $\tau(n)$ (number of divisors), $\omega(n)$ (distinct prime divisors)
- factorization of n: factorizing single number, precomputing all numbers factorization

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Building block: sieve of Eratosthenes

Claim:

Sieve of Eratosthenes to rule them all.

Sieve of Eratosthenes - vanilla

```
vector<bool> isprime(n,true);
for(int i=2;i<n;i++)
    if( isprime[i] )
        for(int j=2*i;j<n;j+=i)
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How to extract factorization of numbers?

Factorization

Idea:

For every number, store ONE of prime divisors.

$$\text{factor}(n) = \text{divisor}(n) \cdot \text{factor}\left(\frac{n}{\text{divisor}(n)}\right)$$

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	2	3	2	5	3	7	2	3	5	11	3	13	2	5	2	17	3	19	5	3	2	23	3	5

```
vector<int> divisor(n);
for(int i=2;i<n;i++)
    if( divisor[i]==0 )
        for(int j=i;j<n;j+=i)
            divisor[j] = i;
```

Observe:

We are actually computing THE LARGEST prime factor.

Extending factorization

Let's precompute $\tau(n)$ (number of divisors).

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```
vector<int> cnt(n);
for(int i=2;i<n;i++)
    if( divisor[i]==i )
        cnt[i] = 1;
    else {
        int j = i/divisor[i];
        if( divisor[i]==divisor[j] )
            cnt[i] = cnt[j]+1;
        else
            cnt[i] = 1;
    }
```

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```
vector<int> tau(n);
for(int i=2;i<n;i++)
    if( divisor[i]==i ){
        tau[i] = 2;
    } else {
        int j = i/divisor[i];
        tau[i] = tau[j]/cnt[i] * (cnt[i]+1);
    }
```

Exercise session

Hashing

Hashing

String A, queries:
is $A[i..i+\ell] = A[j..j+\ell]$?

example:

$A =$

b	a	n	a	n	a	n	a

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$A[2..5] = \texttt{anan}$ $A[4..7] = \texttt{anan}$

YES

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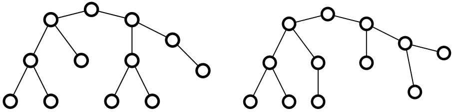

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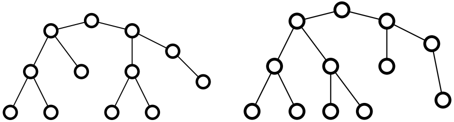

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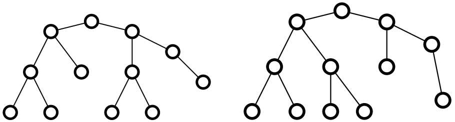

NO

IDEA:

hash objects, use recursive structure
objects = substrings, subtrees

Subwords - Karp Miller Rosenberg algorithm

$A =$

b	a	n	a	n	a	n	a	f

Subwords - Karp Miller Rosenberg algorithm

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b

a

n

a

n

a

n

a

f

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b	a
a	a
n	a
a	a
n	b
a	f
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a	n
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Subwords - Karp Miller Rosenberg algorithm

$A =$

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b	a	a = 1
a	a	b = 2
n	a	f = 3
a	a	n = 4
n	b	
a	f	
n	n	
a	n	
f	n	

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$A =$ | b | a | n | a | n | a | n | a | f |

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$\texttt{anaf} = (\texttt{an}, \texttt{af}) = (6, 5)$
$\texttt{anan} = (\texttt{an}, \texttt{an}) = (6, 6)$
$\texttt{anan} = (\texttt{an}, \texttt{an}) = (6, 6)$
$\texttt{bana} = (\texttt{ba}, \texttt{na}) = (7, 8)$
$\texttt{nana} = (\texttt{na}, \texttt{na}) = (8, 8)$
$\texttt{nana} = (\texttt{na}, \texttt{na}) = (8, 8)$

$\texttt{a} = 1$
$\texttt{b} = 2$
$\texttt{f} = 3$
$\texttt{n} = 4$
$\texttt{af} = 5$
$\texttt{an} = 6$
$\texttt{ba} = 7$
$\texttt{na} = 8$

$\texttt{anaf} = 9$
$\texttt{anan} = 10$
$\texttt{bana} = 11$
$\texttt{nana} = 12$

Subwords - Karp Miller Rosenberg algorithm

$A =$ | b | a | n | a | n | a | n | a | f |

$\texttt{anananaf} = (\texttt{anan}, \texttt{anaf}) = (10, 9)$

$\texttt{bananana} = (\texttt{bana}, \texttt{nana}) = (11, 12)$

a	= 1	anaf	= 9
b	= 2	anan	= 10
f	= 3	bana	= 11
n	= 4	nana	= 12
af	= 5	anananaf	= 13
an	= 6	bananana	= 14
ba	= 7		
na	= 8		

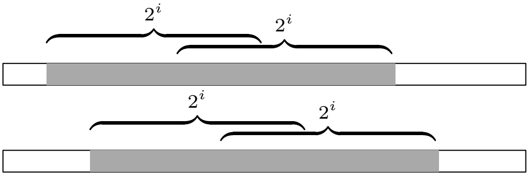
Subwords - Karp Miller Rosenberg algorithm

$A =$

b	a	n	a	n	a	n	a	f

KMR =

2	1	4	1	4	1	4	1	3
7	6	8	6	8	6	8	5	
11	10	12	10	12	9			
14	13							



a = 1
b = 2
f = 3
n = 4
af = 5
an = 6
ba = 7
na = 8

anaf = 9
anan = 10
bana = 11
nana = 12
anananaf = 13
bananana = 14

Pseudocode

```
cnt = 0;
vector<pair<char,int> > letter_position(n);
for(int i=0;i<n;i++)
    letter_position[i] = make_pair(A[i],i);
sort(letter_position.begin(),letter_position.end());
for(int i=0;i<n;i++)
{
    if(i==0 || letter_position[i].first != letter_position[i-1].first)
        cnt++;
    KMR[letter_position[i]][0] = cnt;
}

for(j=1; (1<<j) <= n; j++)
{
    vector<pair<pair<int,int>,int> > pair_position(n);
    for(int i=0;i<n;i++)
        pair_position[i] = make_pair(make_pair(KMR[i][j-1],KMR[i+(1<<j-1)][j]),i);
    sort(pair_position.begin(),pair_position.end());
    for(int i=0;i+(1<<j)<=n;i++)
    {
        if(i==0 || pair_position[i].first != pair_position[i-1].first)
            cnt++;
        KMR[pair_position[i]][j] = cnt;
    }
}
```

Hashing

KMR:

- time complexity $\mathcal{O}(n \log^2 n)$ or $\mathcal{O}(n \log n)$ - too slow
- memory usage $\mathcal{O}(n \log n)$ - too large
- computes "too much"

Testing identity of objects: find hash function h, that:

- $h(X) = h(Y)$ when $X \sim Y$
- $h(X) \neq h(Y)$ when $X \nsim Y$ (hopefully)
- $h(X)$ can be computed using recursive structure of X

Hashing

Required:

- $h(X) = h(Y)$ when $X \sim Y$
- $h(X) \neq h(Y)$ when $X \nsim Y$ (hopefully)
- $h(X)$ can be computed using recursive structure of X

Hashing

Required:

- $h(X) = h(Y)$ when $X \sim Y$
- $h(X) \neq h(Y)$ when $X \nsim Y$ (hopefully)
- $h(X)$ can be computed using recursive structure of X

- Fix large prime p
- $h(A[1..2^i]) = \left(h(A[1..2^{i-1}]) + x_i \cdot h(A[2^{i-1}+1..2^i])\right) \bmod p$,
- where x_i is picked randomly and depends only on i

Subwords - hashing

$A =$

b	a	n	a	n	a	n	a	f

$p = 1009$

Subwords - hashing

$A =$ | b | a | n | a | n | a | n | a | f | $\quad p = 1009$

b	a	n	a	n	a	n	a	f

$$h(\texttt{b}) = 2$$
$$h(\texttt{a}) = 1$$
$$h(\texttt{n}) = 14$$
$$h(\texttt{a}) = 1$$
$$h(\texttt{n}) = 14$$
$$h(\texttt{a}) = 1$$
$$h(\texttt{n}) = 14$$
$$h(\texttt{a}) = 1$$
$$h(\texttt{f}) = 6$$

Subwords - hashing

$A =$ | b | a | n | a | n | a | n | a | f | $\quad p = 1009$

$h(\texttt{b}) = 2$	$h(\texttt{ba}) = 2 + 10 * 1 = 12$
$h(\texttt{a}) = 1$	$h(\texttt{an}) = 1 + 10 * 14 = 141$
$h(\texttt{n}) = 14$	$h(\texttt{na}) = 14 + 10 * 1 = 24$
$h(\texttt{a}) = 1$	$h(\texttt{an}) = 1 + 10 * 14 = 141$
$h(\texttt{n}) = 14$	$h(\texttt{na}) = 14 + 10 * 1 = 24$
$h(\texttt{a}) = 1$	$h(\texttt{an}) = 1 + 10 * 14 = 141$
$h(\texttt{n}) = 14$	$h(\texttt{na}) = 14 + 10 * 1 = 24$
$h(\texttt{a}) = 1$	$h(\texttt{af}) = 1 + 10 * 6 = 61$
$h(\texttt{f}) = 6$	

Subwords - hashing

$A =$	b	a	n	a	n	a	n	a	f

$p = 1009$

$$h(\texttt{b}) = 2$$
$$h(\texttt{a}) = 1$$
$$h(\texttt{n}) = 14$$
$$h(\texttt{a}) = 1$$
$$h(\texttt{n}) = 14$$
$$h(\texttt{a}) = 1$$
$$h(\texttt{n}) = 14$$
$$h(\texttt{a}) = 1$$
$$h(\texttt{f}) = 6$$

$$h(\texttt{ba}) = 2 + {\color{red}10} * 1 = 12$$
$$h(\texttt{an}) = 1 + {\color{red}10} * 14 = 141$$
$$h(\texttt{na}) = 14 + {\color{red}10} * 1 = 24$$
$$h(\texttt{an}) = 1 + {\color{red}10} * 14 = 141$$
$$h(\texttt{na}) = 14 + {\color{red}10} * 1 = 24$$
$$h(\texttt{an}) = 1 + {\color{red}10} * 14 = 141$$
$$h(\texttt{na}) = 14 + {\color{red}10} * 1 = 24$$
$$h(\texttt{af}) = 1 + {\color{red}10} * 6 = 61$$

$$h(\texttt{bana}) = 12 + {\color{red}28} * 24 = 684$$
$$h(\texttt{anan}) = 141 + {\color{red}28} * 141 = 53$$
$$h(\texttt{nana}) = 24 + {\color{red}28} * 24 = 696$$
$$h(\texttt{anan}) = 141 + {\color{red}28} * 141 = 53$$
$$h(\texttt{nana}) = 24 + {\color{red}28} * 24 = 696$$
$$h(\texttt{anaf}) = 141 + {\color{red}28} * 61 = 840$$

Subwords - hashing

$A =$ | b | a | n | a | n | a | n | a | f | $p = 1009$

$h(\texttt{b}) = 2$
$h(\texttt{a}) = 1$
$h(\texttt{n}) = 14$
$h(\texttt{a}) = 1$
$h(\texttt{n}) = 14$
$h(\texttt{a}) = 1$
$h(\texttt{n}) = 14$
$h(\texttt{a}) = 1$
$h(\texttt{f}) = 6$

$h(\texttt{ba}) = 2+10*1 = 12$
$h(\texttt{an}) = 1+10*14 = 141$
$h(\texttt{na}) = 14+10*1 = 24$
$h(\texttt{an}) = 1+10*14 = 141$
$h(\texttt{na}) = 14+10*1 = 24$
$h(\texttt{an}) = 1+10*14 = 141$
$h(\texttt{na}) = 14+10*1 = 24$
$h(\texttt{af}) = 1+10*6 = 61$

$h(\texttt{bana}) = 12+28*24 = 684$
$h(\texttt{anan}) = 141+28*141 = 53$
$h(\texttt{nana}) = 24+28*24 = 696$
$h(\texttt{anan}) = 141+28*141 = 53$
$h(\texttt{nana}) = 24+28*24 = 696$
$h(\texttt{anaf}) = 141+28*61 = 840$

$h(\texttt{ananaf}) = 53+190*840 = 231$
$h(\texttt{bananana}) = 684+190*696 = 745$

Pseudocode

```
for(int i=0;i<n;i++)
     hash[i][0] = A[i]%p;
for(j=1; (1<<j) <= n; j++)
{
     x = 1+rand()%(p-1);
     for(int i=0;i+(1<<j)<=n;i++)
          hash[i][j] = (hash[i][j-1] + x*hash[i+(1<<j-1)][j-1])%p;
}
```

Pseudocode

```
for(int i=0;i<n;i++)
    hash[i][0] = A[i]%p;
for(j=1; (1<<j) <= n; j++)
{
    x = 1+rand()%(p-1);
    for(int i=0;i+(1<<j)<=n;i++)
        hash[i][j] = (hash[i][j-1] + x*hash[i+(1<<j-1)][j-1])%p;
}
```

hashing:

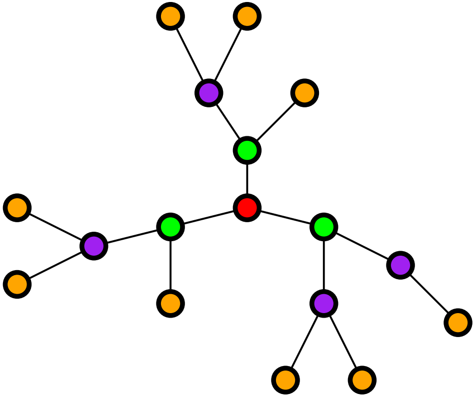
- time complexity $\mathcal{O}(n \log n)$
- much "nicer" memory access – much faster
- memory usage $\mathcal{O}(n \log n)$ - too large
- randomized (small probability of collision)

There is even simpler way of hashing, with $\mathcal{O}(n)$ time and memory, and $\mathcal{O}(1)$ queries. See: Karp-Rabin fingerprints.

Tree isomorphism - labeling

Idea:

- go bottom-up level by level
- label subtree roots

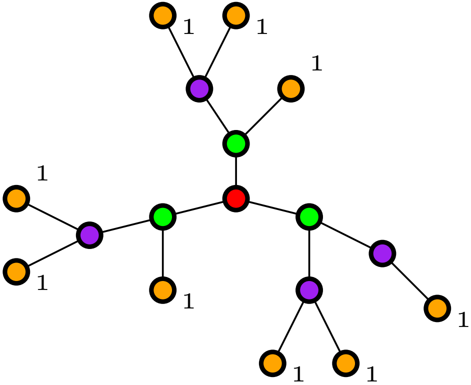


Tree isomorphism - labeling

Idea:

- go bottom-up level by level
- label subtree roots

$$1 = \{\}$$

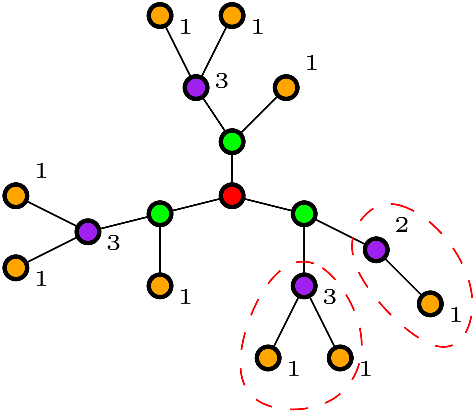


Tree isomorphism - labeling

Idea:

- go bottom-up level by level
- label subtree roots

$$1 = \{\}$$
$$2 = \{1\}$$
$$3 = \{1, 1\}$$

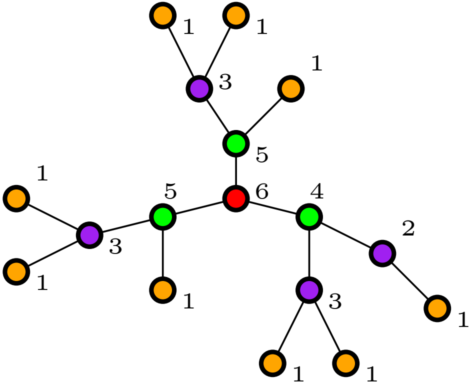


Tree isomorphism - labeling

Idea:

- go bottom-up level by level
- label subtree roots

$$1 = \{\}$$
$$2 = \{1\}$$
$$3 = \{1, 1\}$$
$$4 = \{2, 3\}$$
$$5 = \{1, 3\}$$
$$6 = \{4, 5, 5\}$$



Tree isomorphism - labeling

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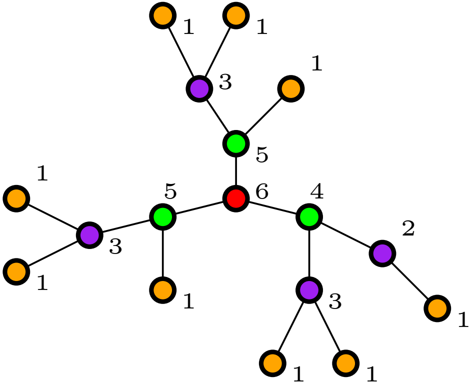

$$1 = \{\}$$
$$2 = \{1\}$$
$$3 = \{1, 1\}$$
$$4 = \{2, 3\}$$
$$5 = \{1, 3\}$$
$$6 = \{4, 5, 5\}$$

Hint:

for each node, sort its children list

Tree isomorphism - hashing

Idea:

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Tree isomorphism - hashing

Idea:

- go bottom-up level by level
- hash subtree roots

Solution:

sort $t_1, t_2, ..$, for node at level i:

$h(\{t_1, t_2, ..\}) =$

$h(t_1) + h(t_2) \cdot X_i + h(t_3) \cdot X_i^2 + ..$

Tree isomorphism - hashing

Idea:

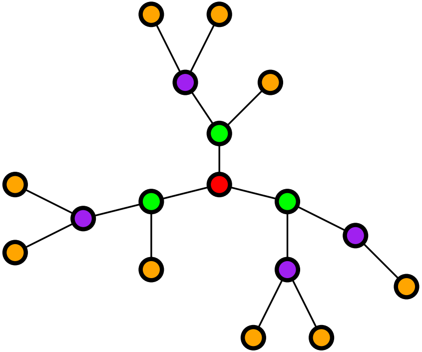
- go bottom-up level by level
- hash subtree roots

Solution:

sort $t_1, t_2, ..,$ for node at level i:
$h(\{t_1, t_2, ..\}) =$
$h(t_1) + h(t_2) \cdot X_i + h(t_3) \cdot X_i^2 + ..$

Careful:

Different node symmetry (mirror-invariant, rotation-invariant, reordering-invariant) require different $h()$.



Tips&Tricks

Tips&Tricks

Tip 1

Learn your tools.

- `bash` syntax
- `grep`
- `head`, `tail`
- `wc`
- `bc`
- `factor`

Tips&Tricks

Tip 1

Learn your tools.

Try:

```
> bc -l
scale = 1000
a(1)*4
e(1)
```

- bash syntax
- `grep`
- `head`, `tail`
- `wc`
- `bc`
- `factor`

Tips&Tricks

Tip 2

Learn language and library.

- What is the memory usage of empty `vector<>`?

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- What does `while(cin >> x)` do?
- `for(auto& i : v)` construction.

Tips&Tricks

Tip 2

Learn language and library.

- What is the memory usage of empty `vector<>`?
- What does `while(cin >> x)` do?
- `for(auto& i : v)` construction.
- `stringstream`

Tips&Tricks

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- What is the memory usage of empty `vector<>`?
- What does `while(cin >> x)` do?
- `for(auto& i : v)` construction.
- `stringstream`
- `vector<>` and `g++ -O2` vs. `g++ -O0`

Tips&Tricks

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- What's wrong with: `for(int i = 0; i < x.size()-1; i++)`?

Tips&Tricks

Tip 2

Learn language and library.

- What is the memory usage of empty `vector<>`?
- What does `while(cin >> x)` do?
- `for(auto& i : v)` construction.
- `stringstream`
- `vector<>` and `g++ -O2` vs. `g++ -O0`
- What's wrong with: `for(int i = 0; i < x.size()-1; i++)`?
- `g++ -ftrapv`

Tips&Tricks

Tip 3

Keep having fun!

Exercise session