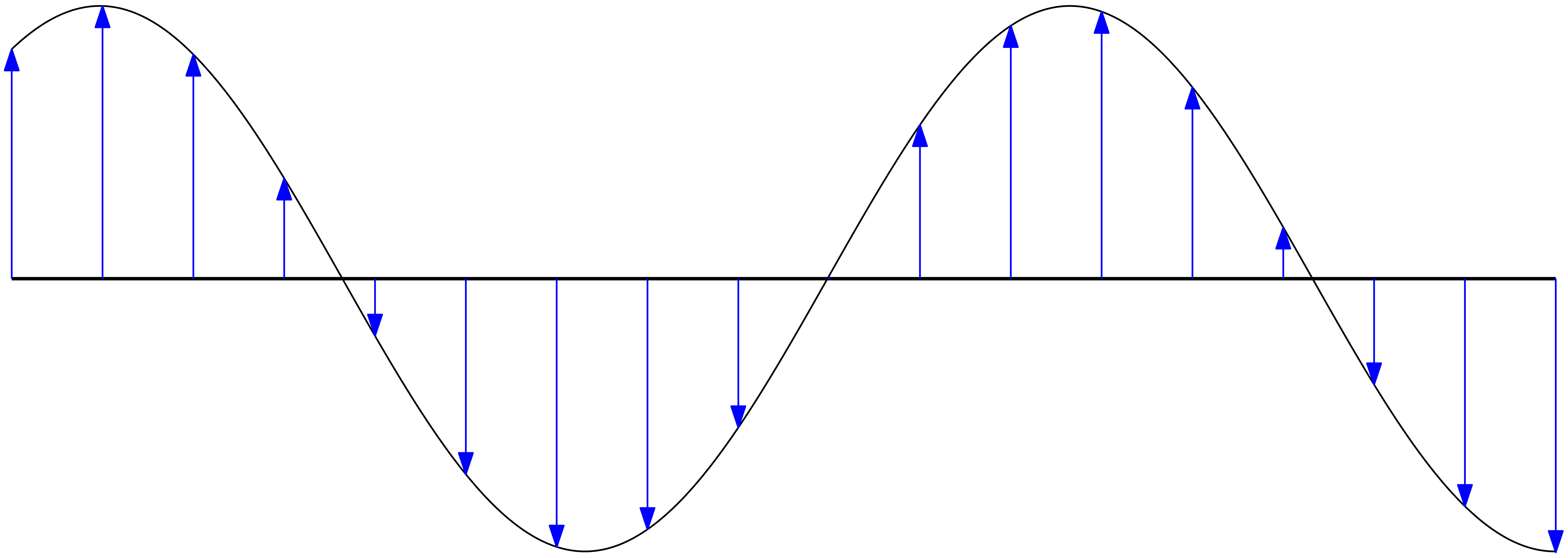
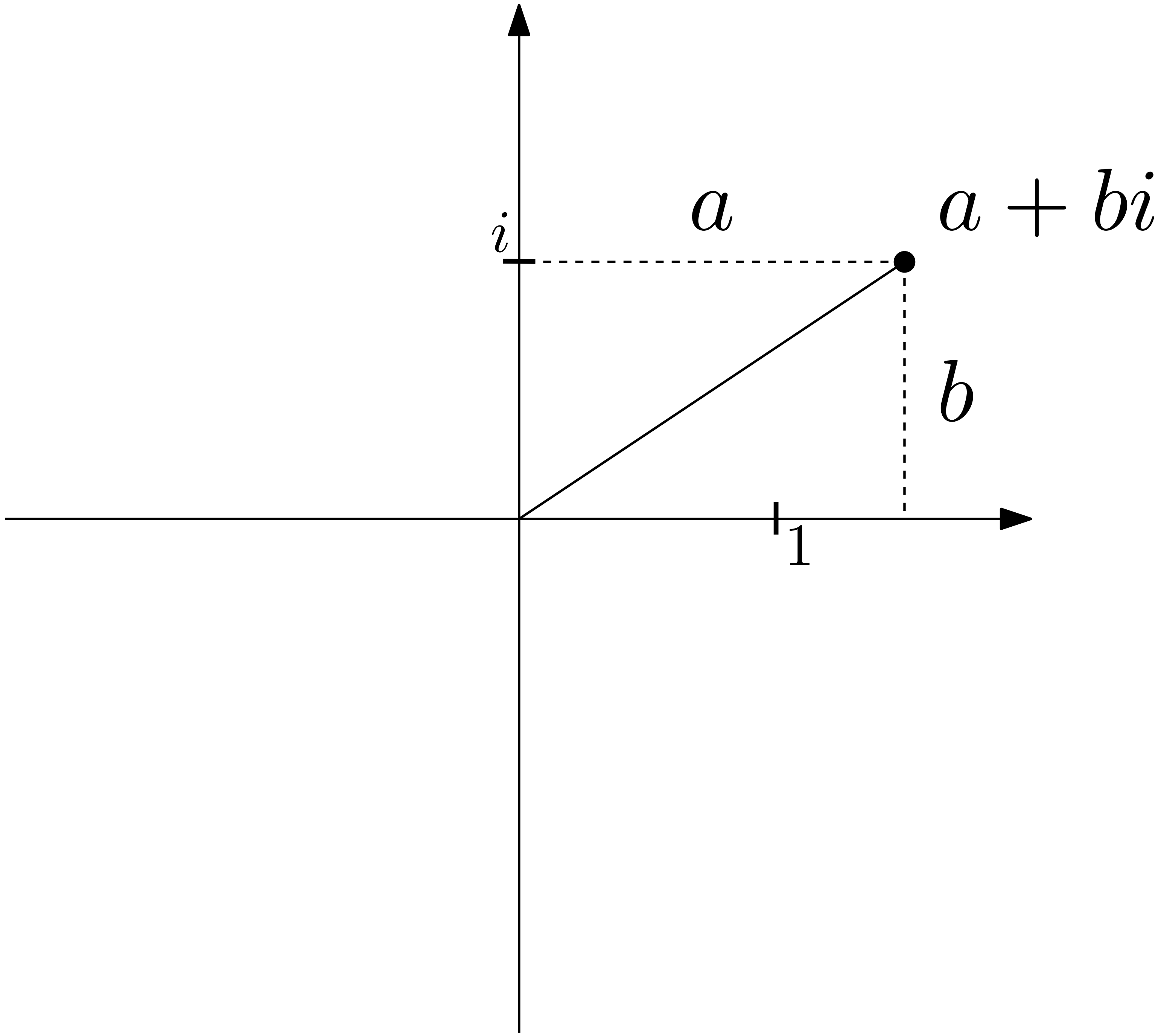
FFT

Algoritmikerho 21.3.

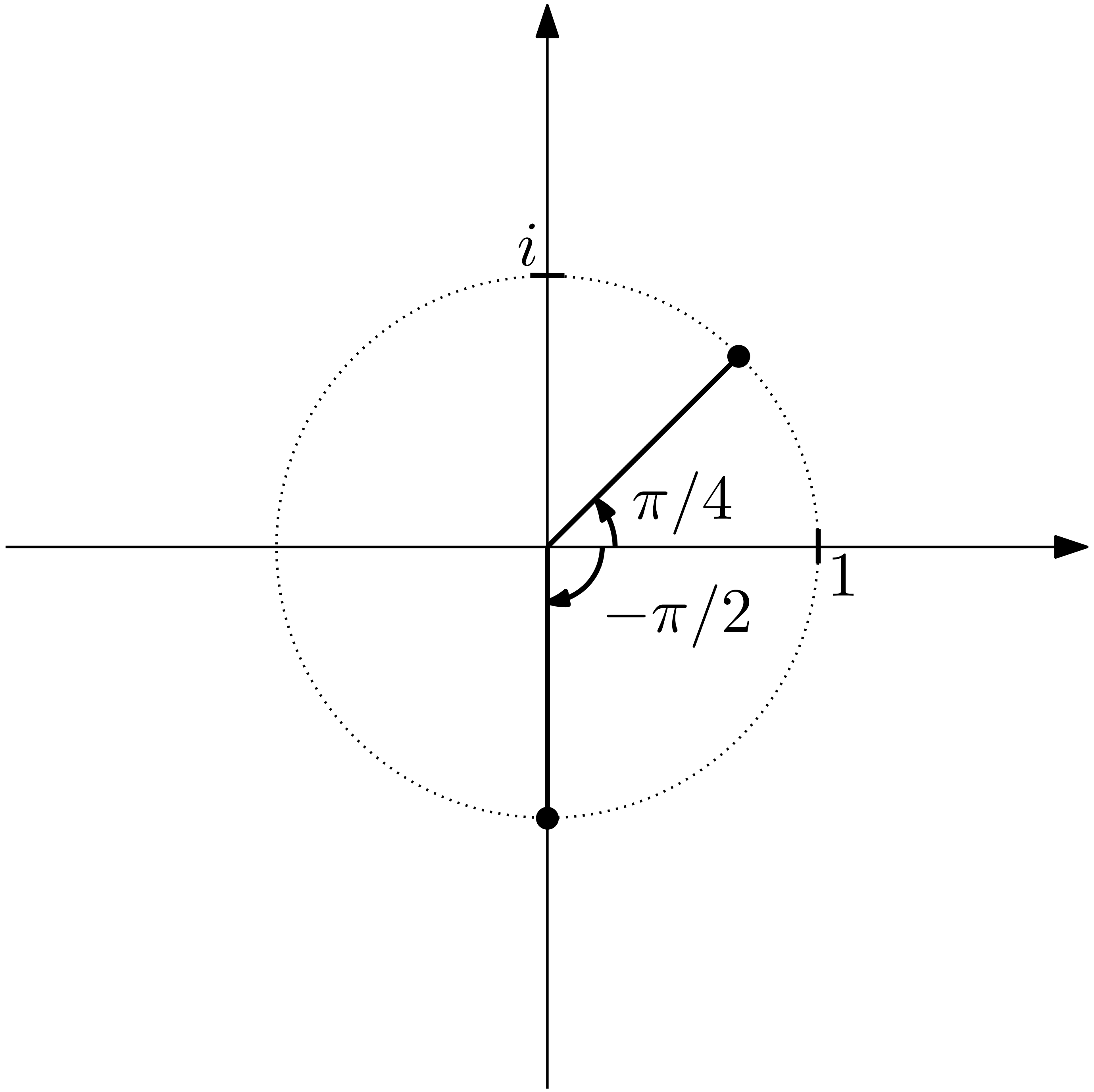
Äänisignaali



Kompleksiluvut



Kompleksiluvut

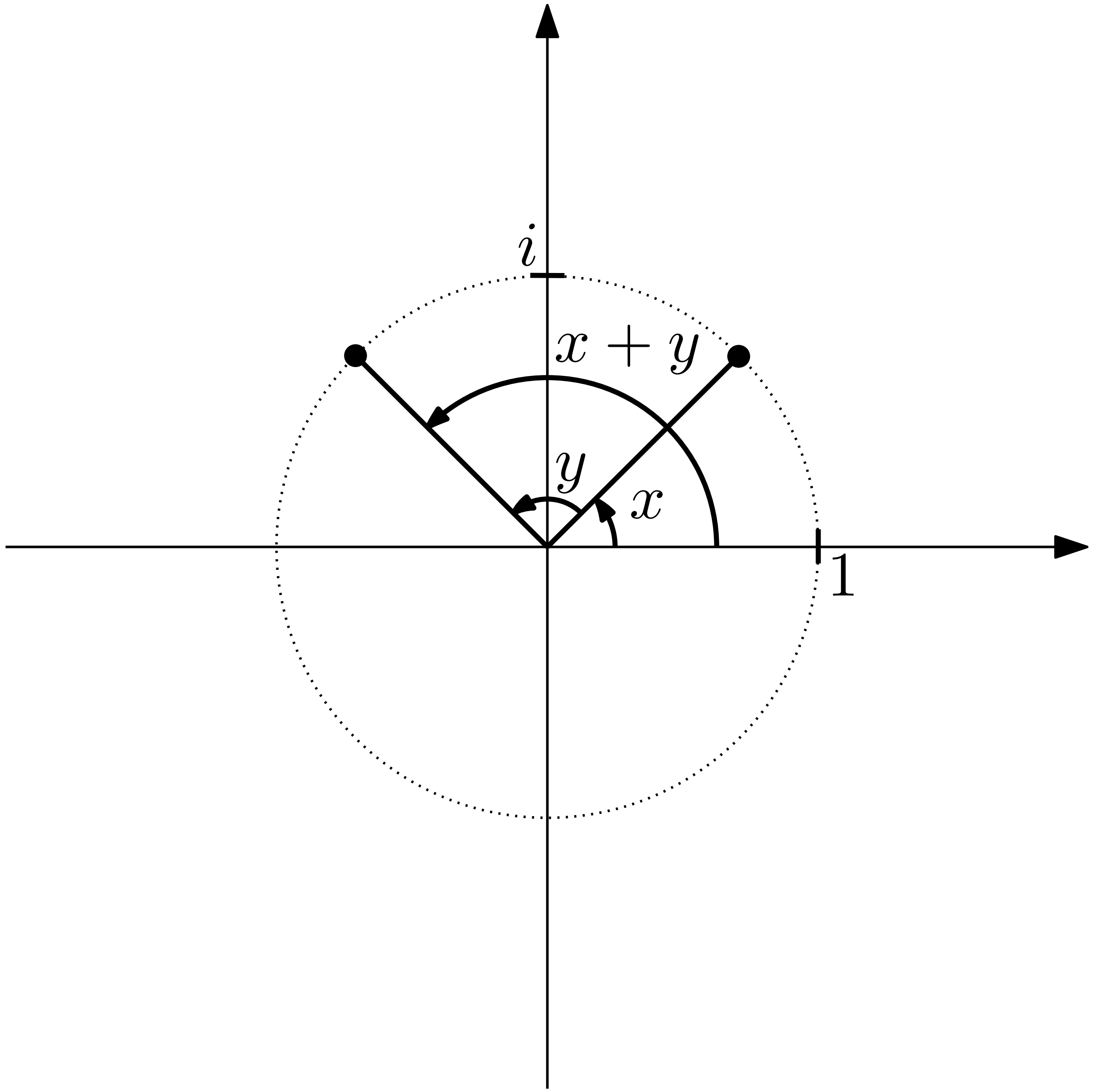


$$e^{ix} = \cos x + i \sin x$$

$$e^{i\pi/4} = 0.707 + 0.707i$$

$$e^{-i\pi/2} = e^{i3\pi/2} = 0 - 1i$$

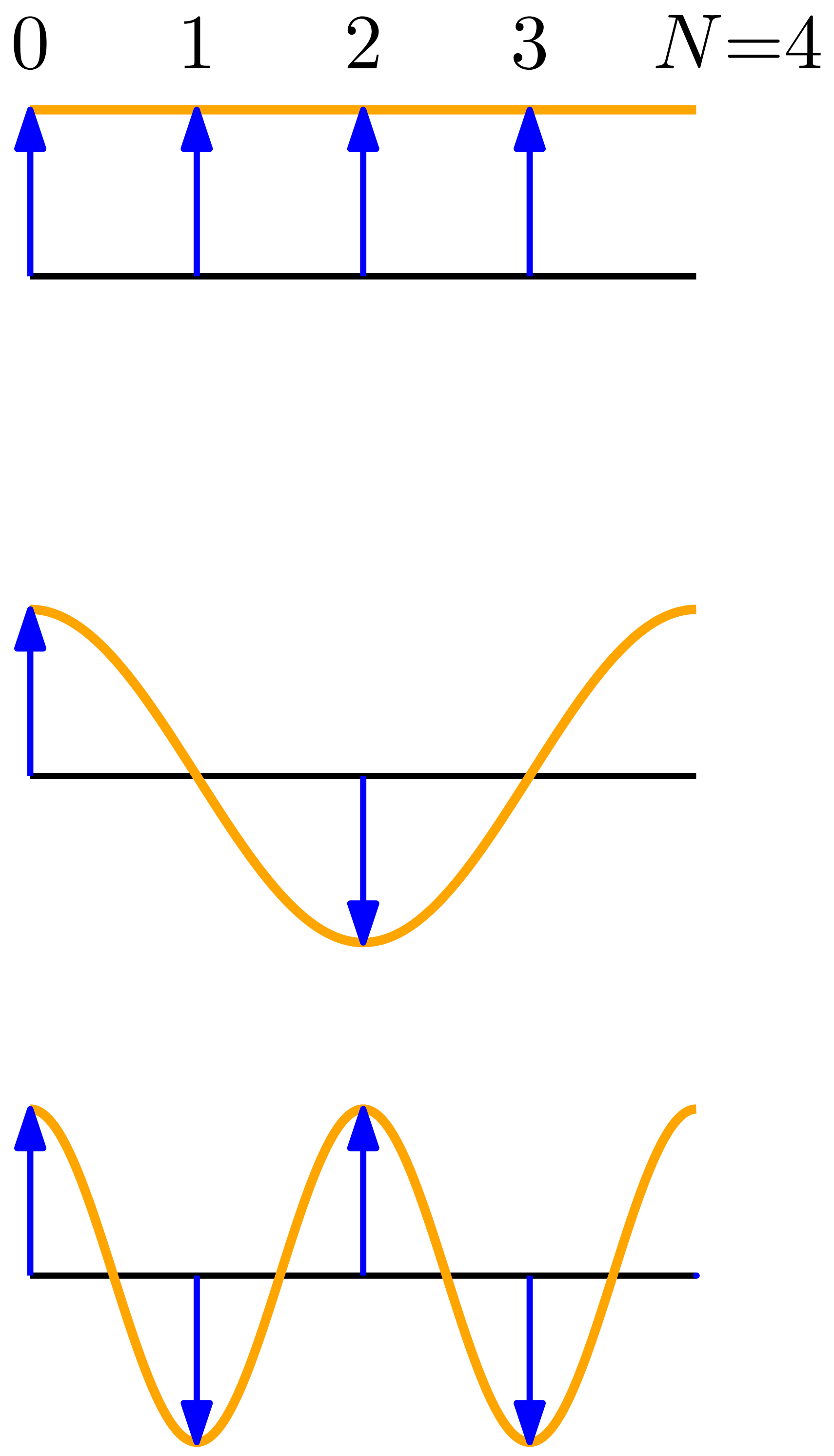
Kompleksiluvut



$$e^{ix} = \cos x + i \sin x$$

$$e^{ix} e^{iy} = e^{i(x+y)}$$

Taajuushajotelma



$$f_0[n] = e^{i2\pi \frac{n}{N} \cdot 0}$$

$$f_1[n] = e^{i2\pi \frac{n}{N} \cdot 1}$$

$$f_2[n] = e^{i2\pi \frac{n}{N} \cdot 2}$$

$$f_3[n] = e^{i2\pi \frac{n}{N} \cdot 3} = e^{-i2\pi \frac{n}{N} \cdot 1} = f_{-1}[n]$$

$$x[n] = af_0[n] + bf_1[n] + cf_2[n] + df_3[n]$$

Diskreetti Fourier-muunnos (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-i2\pi \frac{n}{N} k}$$

Esim. $x = f_1$, eli $x[n] = e^{i2\pi \frac{n}{N}}$

- $X[1] = \sum_{n=0}^{N-1} e^{i2\pi \frac{n}{N} \cdot 1} e^{-i2\pi \frac{n}{N} \cdot 1} = \sum_{n=0}^{N-1} e^{0} = N$
- $X[2] = \sum_{n=0}^{N-1} e^{i2\pi \frac{n}{N} \cdot 1} e^{-i2\pi \frac{n}{N} \cdot 2} = \sum_{n=0}^{N-1} e^{-i2\pi \frac{n}{N} \cdot 1} = -i - 1 + i + 1 = 0$

Diskreetti Fourier-muunnos (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-i 2\pi \frac{n}{N} k}$$

Voidaan kirjoittaa myös:

$$X[k] = \sum_{n=0}^{N-1} x[n] f_{-k}[n]$$

DFT-matriisi

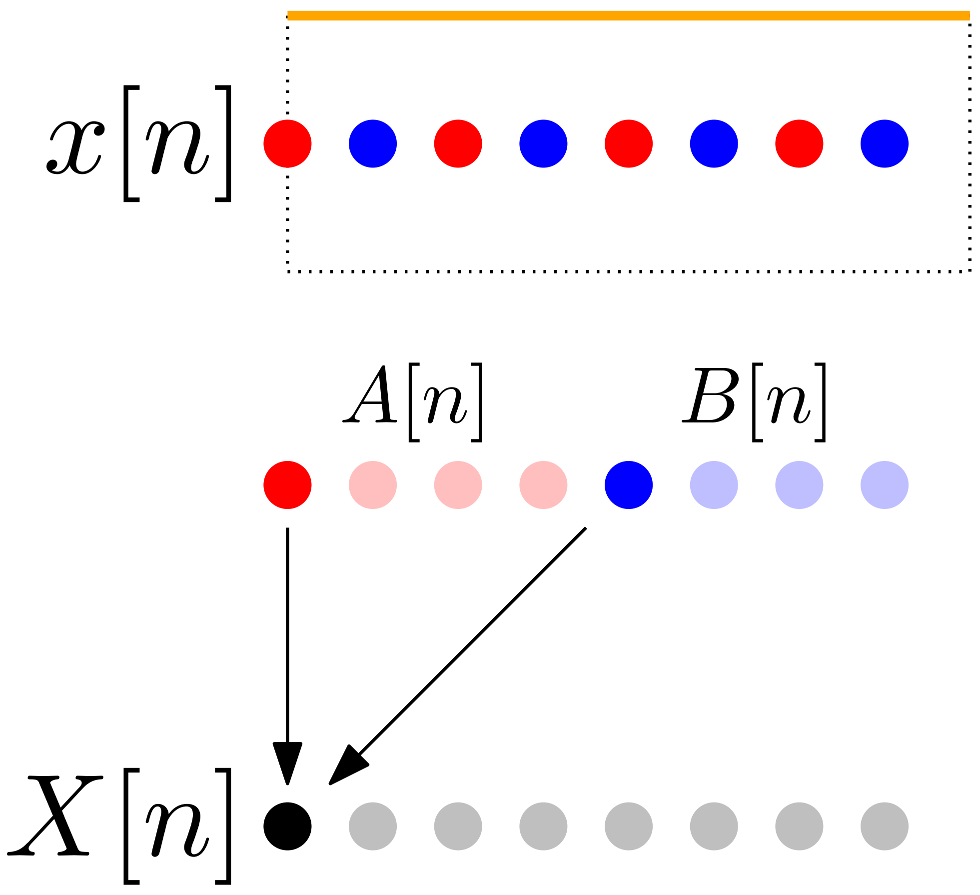
$$\omega = e^{-i2\pi\frac{1}{N}}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^8 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$$

Nopea Fourier-muunnos (FFT)

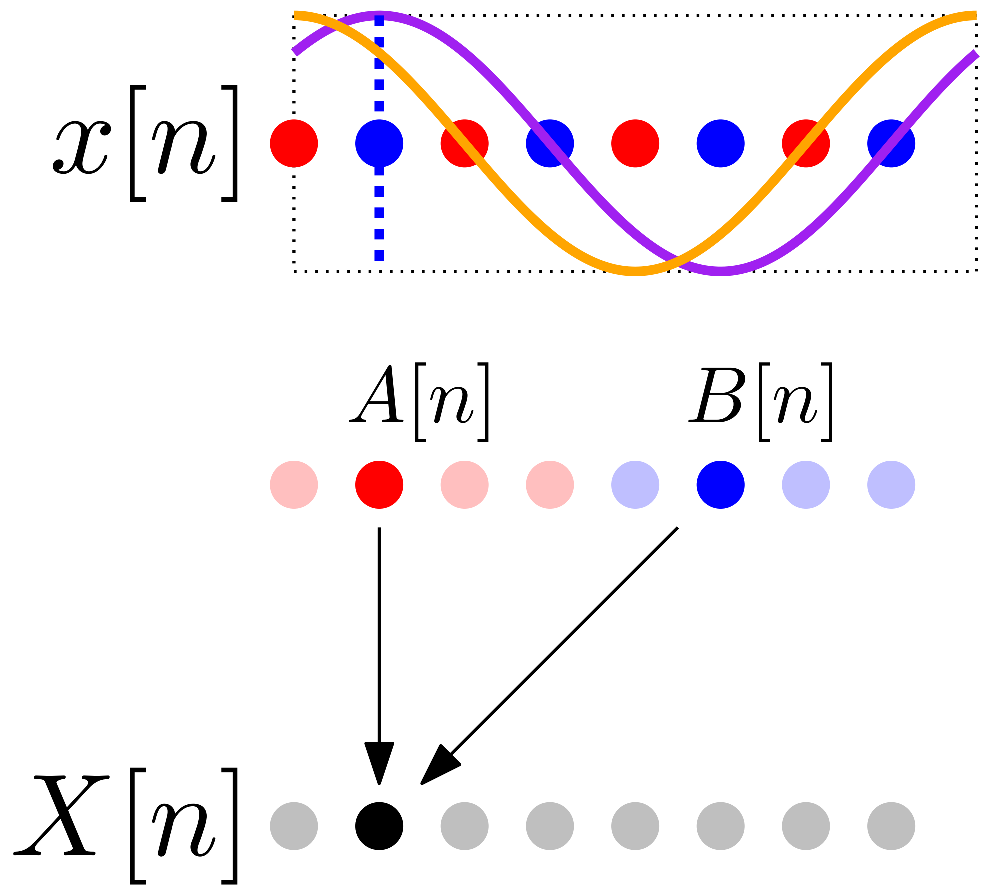
- Tapa laskea DFT ajassa $O(N \log N)$ (vrt. $O(N^2)$)
- Cooley–Tukey-menetelmä, hajota ja hallitse
- DFT taulukolle x saadaan laskemalla ensin DFT:t taulukoille $a[n] = x[2n]$ ja $b[n] = x[2n+1]$ (joka toinen).
- N oltava kahden potenssi

Nopea Fourier-muunnos (FFT)



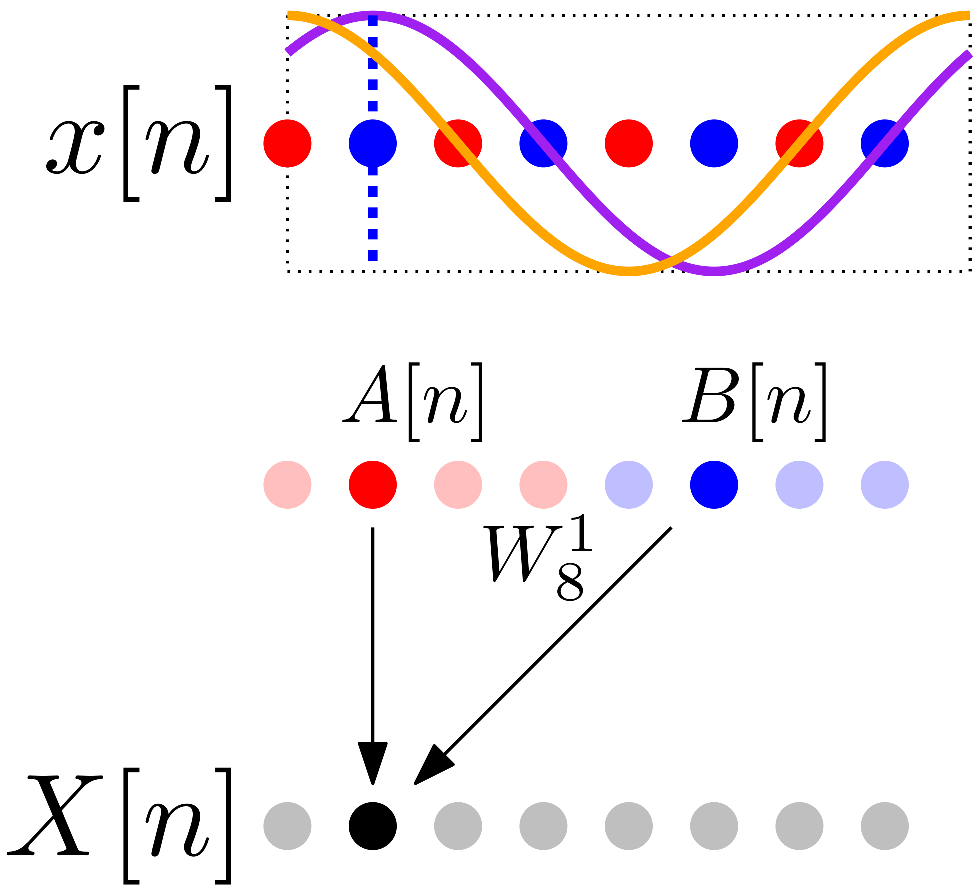
$$X[0] = A[0] + B[0]$$

Nopea Fourier-muunnos (FFT)



$$X[1] = A[1] + B[1]e^{-i2\pi\frac{1}{N}\cdot 1}$$

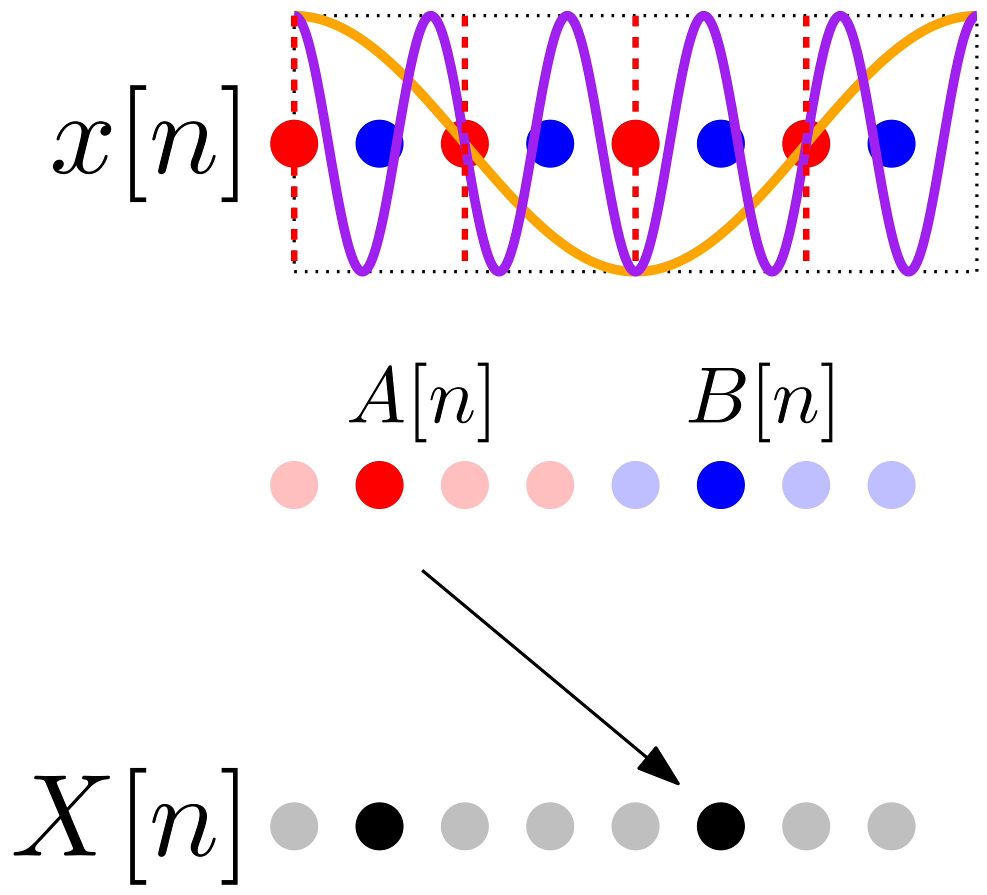
Nopea Fourier-muunnos (FFT)



$$W_N = e^{-i2\pi \frac{1}{N}}$$

$$X[1] = A[1] + B[1] W_8^1$$

Nopea Fourier-muunnos (FFT)

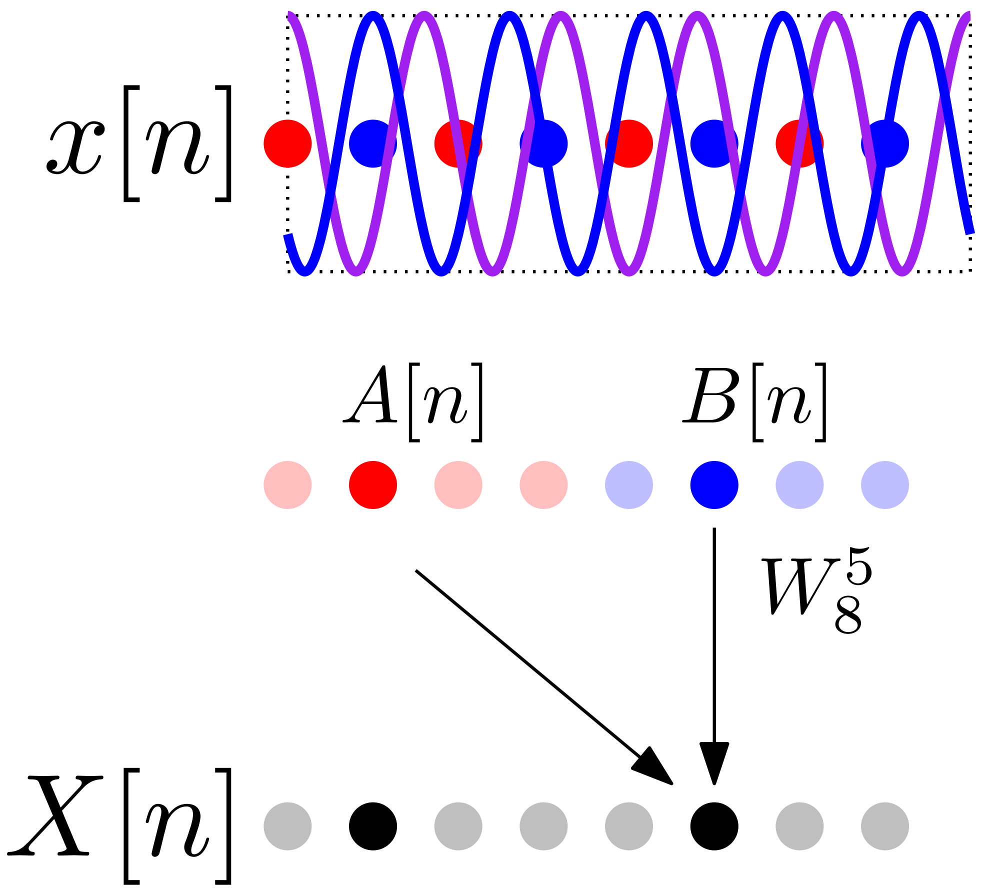


$$W_N = e^{-i2\pi\frac{1}{N}}$$

$$X[5] = A[1] +$$

Kun $N = 4,\ f_1 = f_5$

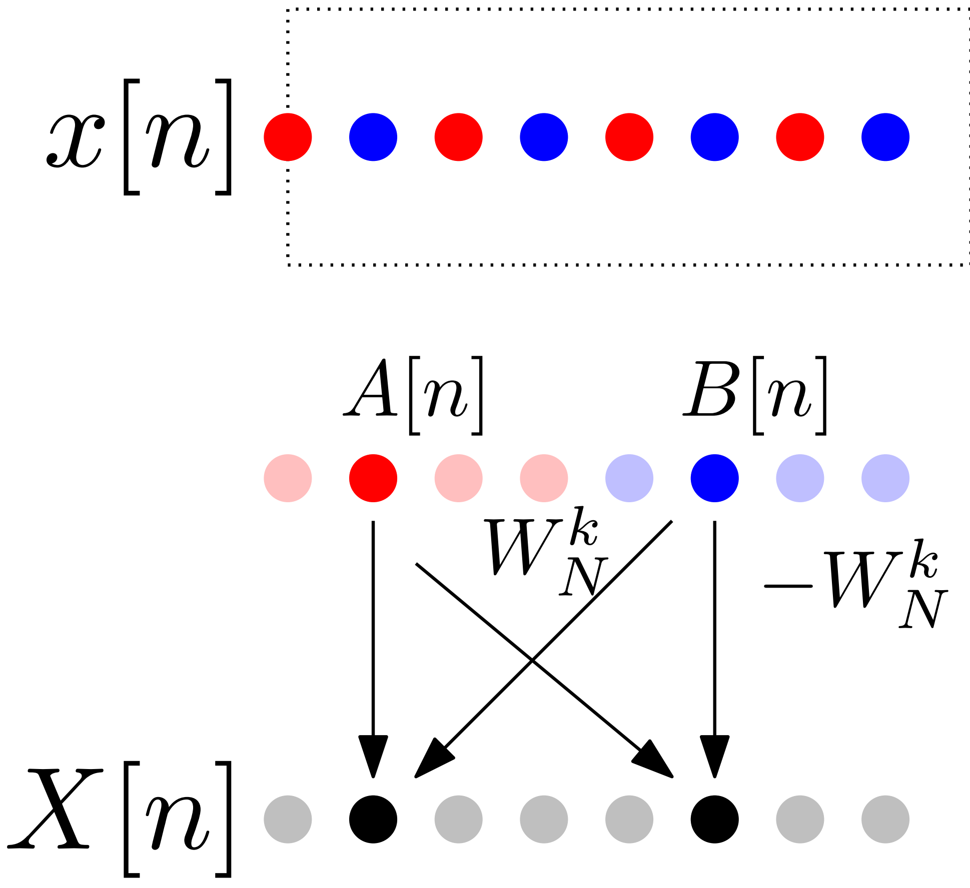
Nopea Fourier-muunnos (FFT)



$$W_N = e^{-i2\pi\frac{1}{N}}$$

$$X[5] = A[1] + B[1]W_8^5$$

Nopea Fourier-muunnos (FFT)



$$W_N = e^{-i2\pi\frac{1}{N}}$$

$$X[k] = A[k] + B[k]W_N^k$$

$$X[k+\tfrac{N}{2}] = A[k] - B[k]W_N^k$$

$$W_N^{k+\frac{N}{2}} = W_N^k W_N^{\frac{N}{2}} = -W_N^k$$